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INFLUENCE OF OPACITY ON THE PULSATIONAL STABILITY OF MASSIVE STARS WITH UNIFORM CHEMICAL COMPOSITION. II. MODIFIED KRAMERS OPACITY

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ABSTRACT

The pulsational stability of massive, chemically homogeneous stars of Population I has been investigated for a range of simple opacity representations consisting of a straight sum of electron-scattering and modified Kramers opacity. The critical mass for stability against nuclear-energized pulsations is found to be extraordinarily sensitive to small changes in the coefficient and/or temperature exponent of the absorption part of the opacity law. A gradual increase in the atomic absorption (dominant near the stellar surface) first raises the critical mass, then restricts the upper mass limit for instability, and finally eliminates pulsational instability at all masses.

Subject headings: massive stars — opacities — pulsation

I. INTRODUCTION

Stellar models of sufficiently high mass have always been found to be unstable against nuclear-energized pulsations. Among the factors influencing the pulsational instability of such models, one of the most poorly determined is the opacity. By adopting Kramers's law of opacity, Ledoux (1941) determined the *critical mass* dividing stable from unstable models to be $\sim 300\ M_{\odot}$ (for a modern Population I chemical composition). Schwarzschild and Härm (1959) later redetermined the critical mass by using only electron scattering and found $\sim 60\ M_{\odot}$. Nevertheless, the outermost layers of the stars are greatly affected by atomic absorption; and, not surprisingly, the critical mass rises to $\sim 100\ M_{\odot}$ when the full Cox-Stewart (1965) opacities are adopted (Stothers and Simon 1970, "Paper I"). The Cox-Stewart opacities are based on a "hydrogenic" model of the atom; but a switch to the "Thomas-Fermi" atomic model has been shown to alter the opacities rather seriously, mostly by increasing them (Carson, Mayers, and Stibbs 1968).

Since definitive opacities are lacking at the present time, it is of interest to investigate the behavior of the critical mass for pulsational stability for a range of simple, though apparently realistic, opacity representations. In this way, we can learn how sensitive the critical mass really is to atomic absorption.

II. CRITICAL MODELS

With the exception of the selected opacity representation, the same input physics as in Paper I has been used to construct the present equilibrium and pulsational models. To determine pulsational stability or instability, the linearized quasi-adiabatic approximation has again been employed. A (hydrogen, metals) content of $(X, Z) = (0.70, 0.03)$ has been adopted.

Since the form of the nuclear-energy terms, which provide the driving of the pulsations, is taken to be the same in all our models, the derived variation in the critical masses is due strictly to the opacity, which affects the models in two ways. First, a larger opacity, or an opacity that increases more steeply with distance away from the stellar center (i.e., toward lower temperature and density), is found to distend the stellar radius more, and therefore to cause a larger central condensation. But the relative pressure due to radiation is always nearly fixed by the total mass and mean molecular weight of the star. Since a large central condensation stabilizes a star and a high radiation pressure destabilizes it, we find that a low opacity or a high stellar mass leads to greater pulsational instability. These arguments are all well known and need no further elaboration here.

The opacity representations used in the present work are taken to be of the form of a straight sum of electron-scattering and modified Kramers opacity:

$$\kappa = 0.19(1 + X) + \kappa_0 Z[(1 + X)\rho]^{\alpha} T^{-\eta} .$$

We consider two approximate fits to tables of "hydrogenic" opacities computed by Keller and Meyerott (1955), which are not, for the conditions appropriate to massive stars, very different from the newer opacities of Cox and Stewart (1965). The first fit was made by Kushwaha (1957), who found that the unmodified Kramers law gave a fair approximation, viz., $\kappa_0 = 4.34 \times 10^{25}$, $\alpha = 1$, and $\eta = 3.5$. The second fit is due to Larson (Morris and Demarque 1966), who gave, with a slight rearrangement here: $\kappa_0 = 1.58 \times 10^{23}$, $\alpha = 0.67$, and $\eta = 3.21$. Comparison with the actual Keller-Meyerott tables shows that Kushwaha's form provides a rather more accurate fit than does Larson's form at the low densities of stars of interest here. But this is not a really relevant consideration since the tabular values of opacity are themselves uncertain at low density and temperature.

The critical mass for Kushwaha's form is found to be 69 $M_\odot$. Stellar models with masses heavier than this are all pulsationally unstable.[1]

If one recognizes that $\rho \propto T^3$, approximately, in massive stars, then Larson's absorption formula is seen to have a steeper temperature dependence than does Kushwaha's absorption formula. Such a steep temperature dependence causes the central condensation of the models built with Larson's formula to increase with stellar mass faster than the relative radiation pressure does.[2] As a result, no critical mass exists at all when $\kappa_0 = 1.58 \times 10^{23}$! However, the minimum of pulsational stability is found to occur at a finite mass (about 110 $M_\odot$). If the coefficient in the absorption formula is progressively reduced, the temperature dependence of the whole opacity law is weakened. Therefore, the stability of the models is lowered at every mass, although the mass at which stability is least increases. When $\kappa_0 = 0.79 \times 10^{23}$, a true critical mass develops at 300 $M_\odot$; the models are only marginally unstable between 300 and 1800 $M_\odot$, above which they remain stable. For a further reduction of κ_0, the decrease of stability at each mass causes the critical mass to drop somewhat, and the upper mass limit of the unstable models to grow very rapidly. Eventually the situation becomes indistinguishable from that for electron scattering alone ($\kappa_0 = 0$), for which the critical mass is 54 $M_\odot$ with all higher masses being unstable.

[1] Just for comparison, we have also determined that the critical mass is 103 $M_\odot$ based on a set of very detailed formula fits (Iben and Ehrman 1962) to the Keller-Meyerott tables, and 106 $M_\odot$ based on a similarly accurate formula fit (Paper I) to the Cox-Stewart tables.

[2] The steep temperature dependence also enhances the pulsational damping directly through $\delta\kappa/\kappa$, but this effect is less important here than is the increase of central condensation.

III. CONCLUSION

The influence of atomic absorption on the pulsational stability of massive, chemically homogeneous stars of Population I has been reinvestigated for a range of simple opacity representations consisting of a straight sum of electron-scattering and modified Kramers opacity. The critical mass for stability against nuclear-energized pulsations is found to be extraordinarily sensitive to small changes in the coefficient and/or temperature exponent of the absorption part of the opacity law. When the contribution from atomic absorption to the total opacity is increased, the critical mass first rises, with the higher masses remaining unstable (as was already known). Then, with a further increase of atomic absorption, the upper mass limit of the unstable models becomes finite, decreasing rapidly. And, finally, pulsational instability disappears at all masses.

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